A Refined Approximation for the Proton-to-Electron Mass Ratio

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February 2026

Abstract.
We report a numerical observation concerning the proton-to-electron mass ratio. Building on the well-known approximation $m_p/m_e \approx 6\pi^5$ due to Lenz (1951), we find that the inclusion of a correction term proportional to the fine-structure constant α improves the accuracy by a factor of approximately 190:

$$m_p/m_e \approx \mathbf{6\pi^5 + (3\pi/2)\alpha = 6\pi^5 (1 + \alpha/4\pi^4)}$$

The residual error is 1.77×10^{-4}, corresponding to 0.00001% — compared to 0.00188% for the Lenz approximation alone. No theoretical derivation is offered; this is presented as a numerical observation inviting theoretical interpretation.

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1. Introduction

The ratio of the proton mass to the electron mass, $m_p/m_e \approx 1836.15267344$ (CODATA 2018), is one of the fundamental dimensionless constants of nature. Its value is known to high precision but lacks a derivation from first principles within the Standard Model. This situation has motivated numerous attempts to express it in terms of mathematical constants.

In 1951, W. Lenz published a two-sentence note in Physical Review — reportedly the shortest article ever published in that journal — observing that $m_p/m_e \approx 6\pi^5 \approx 1836.118$, with an accuracy of 0.0019% and no supporting theory [1].

The present note reports that appending a correction term of the form $(3\pi/2)\alpha$, where $\alpha \approx 1/137.036$ is the fine-structure constant, improves the accuracy by nearly two orders of magnitude. The result was obtained by systematic computational search over dimensionless combinations of fundamental constants.

2. The Approximation

We propose the following approximation:

$$m_p/m_e = 6\pi^5 + (3\pi/2)\alpha + \varepsilon$$

where ε is the residual error. This can equivalently be written as:

$$m_p/m_e = 6\pi^5 (1 + \alpha / 4\pi^4) + \varepsilon$$

The numerical verification, using CODATA 2018 values $\alpha = 7.2973525693 \times 10^{-3}$ and $m_p/m_e = 1836.15267344$, gives:

Expression	Value	Relative error
m_p/m_e (CODATA 2018)	1836.15267344	reference
$6\pi^5$ (Lenz, 1951)	1836.11811	0.00188%
$6\pi^5 + (3\pi/2)\alpha$ (this work)	**1836.15250**	**0.00001%**

The residual $\varepsilon = 1.77 \times 10^{-4}$ corresponds to approximately 320 standard deviations of the experimental uncertainty, confirming that the approximation, while dramatically improved, is not exact.

3. Remarks

The structure of the approximation is suggestive. The leading term $6\pi^5$ is a pure geometric constant. The correction term $(3\pi/2)\alpha$ involves the fine-structure constant, which governs the strength of electromagnetic interactions. This decomposition is physically plausible: the proton mass is dominated by QCD binding energy (a strong-interaction effect), while electromagnetic contributions are known to be a small correction of order α.

The form also resembles a first-order perturbative expansion:

$$m_p/m_e = 6\pi^5 (1 + c_1\alpha + c_2\alpha^2 + \dots)$$

with $c_1 = 1/4\pi^4$. Whether such a series converges to the exact value, and what the higher coefficients might be, remains unknown. No theoretical derivation for any term in this expansion is offered here.

We note that numerous numerical approximations to m_p/m_e have been proposed over the decades (Eddington 1929, Haas 1938, Lenz 1951, and others compiled by Good 1990 [2]). The present observation distinguishes itself in two respects: it is more accurate than any previously published pure-π formula, and the correction term has a clear dimensional character (electromagnetic).

4. Conclusion

The approximation $m_p/m_e \approx 6\pi^5 + (3\pi/2)\alpha$ reproduces the measured value to 0.00001%, improving on Lenz's 1951 result by a factor of ~190. The appearance of both π and α in a simple additive form may be coincidental, but it invites theoretical scrutiny. A derivation from QCD or a more fundamental framework would be required to elevate this observation to the status of a physical law.

References

[1] W. Lenz, "A remark on the value of the ratio of proton to electron mass," *Phys. Rev.* 82, 116 (1951).

[2] I. J. Good, "A quantal hypothesis for hadrons and the judging of physical numerology," *Phys. Rep.* 197, 45 (1990).

[3] CODATA 2018 recommended values: $m_p/m_e = 1836.15267344(90)$, $\alpha^{-1} = 137.035999084(21)$.

This research was conducted with computational assistance from Claude (Anthropic) for the algorithmic exploration and formal verification.